SET THEORY AND THE SEARCH FOR EXTRATERRESTRIAL LIFE

Probability theory suggests that the universe must be teeming with life. According to UK physicist and TV personality Brian Cox, our galaxy (the Milky Way) counts 20 billion planets in the habitable zones of their stars. And there are 2 trillion galaxies in the universe (give or take a few billion). With this many candidate planets, the universe must be teeming with life, even if the probability of life emerging on any one habitable planet is astronomically small. This is the probabilistic case for extraterrestrial life. But wait a minute. To estimate the number of extraterrestrial civilizations, we must know two things: (1) the number of habitable planets, which we more or less know, and (2) the probability of life emerging on any one planet when all necessary conditions for life are met. This we don't know. Suppose the odds for life emerging are 1/N, where N is the total number of habitable planets in the universe. These odds allow for one living planet only: Earth. But it is highly unlikely that the odds for life are precisely 1/N. These odds must be either much smaller, in which case we're a statistical fluke. Or vastly bigger, suggesting an abundance of intelligent life in the universe. But if that is so, where is everyone? That is the Fermi paradox (Enrico Fermi, 1950).

Our civilization started with the advance of agriculture some 10,000 years ago. Technological progress was slow until the 19th Century but then we took off. From the telegraph (1840s) to the Internet (1990s) took only a century and a half, from first flight (1903) to landing on the moon (1969) just 66 years. Imagine where we'll be a thousand years on. Imagine where an extraterrestrial civilization will be that had a head start on us of a million years, or just a 100,000 – a mere instance on timescale of the universe. This civilization would not only have colonized its own galaxy but the entire universe. But nobody has. Life in the universe had billions of years to develop. With that much time for a (relatively tiny) head start on us, extraterrestrial civilizations should be ubiquitous throughout the universe. But we haven't heard from any of them. A plausible solution to this Fermi paradox is that we haven't found extraterrestrial life because it's just not there. Alternatively, one can argue that extraterrestrial civilizations have already been here and left again, or that we just hae to look better. But none of that really diffuses the Fermi paradox. In sum, probability theory can argue both ways. Probabilistically, the universe may be teeming with life but we may also be alone. Can set theory do any better?

For a set theoretical approach, we have to consider the necessary conditions for life. The website sciencelearn.org.nz suggests 25 necessary elements for life. Among them the Big Four: carbon, oxygen, hydrogen and nitrogen. The Wikipedia page "Biological roles of the elements" suggests that 19 of the 116 elements are necessary for life and that a further 17 are necessary for at least some forms of life. So if we want to be certain of sufficiency, we need all 36 elements. Which suggests that the configuration explaining life (on our planet) consists of 36 conditions. That's seriously complex causality. But it's not all. Among others, the elements must be combined into liquid water and carbohydrates to enable life. So we now have 38 conditions. And we're still not there. Our solar system is weird in that it has planets of different sizes. Earth size planets like Mars and Venus but also giants like Jupiter. The solar systems we have thus far discovered seem to mostly have planets of a fairly uniform size. The thing is, Jupiter size planets are too big to sustain life (their gravity is too strong). However, we need those giants to catch asteroids and other space debris that would otherwise have spoilt the primordial soup from which life on earth emerged. Earth itself is weird because has a tilted axis. It gives our planet seasons and this seems to have played a role in the emergence of life. Luckily, the tilt is small otherwise we'd have had too extreme seasons. Earth also has a big moon which keeps the tilt of Earth's axis fairly constant. Without such a big moon, Earth would spin erratically. The rotation speed of Earth is important because it allows photosynthesis. Earth has a liquid core that produces the magnetic field that protects us from cosmic rays and solar

flares. It is also responsible for vulcanism, which played a role in producing the primordial soup from which life emerged. This suggests six further conditions that need to be added to the configuration explaining life: (1) the presence of giant planets, (2) a slightly tilted axis, (3) a big moon, (4) a specific rotation speed, (5) magnetism and (6) vulcanism. And even then we're not there. When all necessary conditions for life are present, it still requires (7) a stroke of luck for life to emerge from the primordial soup. So we now have a configuration of 45 conditions (38 elements plus the above 7) to explain life. As every condition may be present or absent, these 45 conditions can occur in nearly 35.2 billion possible combinations. However, as far as we know, in our galaxy, only one of those configurations describes a planet (Earth) where life emerged. The rest of the planets are described by configurations that do not allow life to emerge. Of course, there are plenty more planets in other galaxies. But with life possible in only 1 of 35.2 billion possible combinations of conditions for life, there is a strong possibility that life only ever emerged on just a single planet, Earth.

Let me support my argument with an example of more earthly dimensions. Suppose I want to describe myself uniquely out of a population of 8 billion people. I could say that I am (1) male, (2) white, (3) married and (4) interracially so, (5) a daddy, (6) a PhD, (7) have a personal website, (8) live in a city, (9) own a semi-detached house and a (10) sizeable car. This configuration does a good job of describing me, but I would not be at all surprised if it described tens of thousands of others too, if not more. In fact, I know some of them. But now consider the following configuration:

(1) I live in Tilburg.
(2) I have published in Sociological Methods & Research.
(3) On a set theoretical method (QCA).
(4) I'm an Elvis fan.
(5) I know the national anthem of a country that no longer exists (East Germany, don't ask me why).

I am pretty confident that already the first three conditions uniquely describe me. Just the first condition narrows set membership down from 8 billion to 220,000 people. Add the fourth and fifth conditions and the configuration is guaranteed to cover only me. What's the difference between the first and the second configurations? The first configuration contains ten conditions that are all fairly general characteristics. Each condition describes a very large number of people. So it's almost impossible for an intersection of those conditions to define a nearly empty set. The second configuration has five conditions that are all much more specific. Each of those describes a (very) much smaller set of people. Consequently, the intersection of those sets is guaranteed to describe a very sparsely populated set. One that includes only me. The important point is that the above 45 necessary conditions for life seem to be very specific conditions. Very few planets have slightly tilted axis, very few have liquid water, etc. Set theory states that the more complex a configuration, the fewer cases it will cover. When individual conditions already cover a small number of cases each, a configuration of those conditions will quickly define a nearly empty set. If only three such conditions reduce a population from 8 billion to just a single person, it lies firmly within reason that 45 specific characteristics may describe just a single planet in the entire universe.

Of course, this doesn't prove that Earth is the only living planet in the universe. However, while probability theory does not firmly point towards either the presence or absence of extraterrestrial life, set theory very clearly does. The sheer complexity of the causality required to explain life makes a very compelling argument that we're alone.

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